4 Solver MEBDFDAE

4.1 General information

Author:	J. Cash
first version:	November , 1998
last update:	February, 2006
language:	Fortran 77
availability:	the code MEBDFDAE is freely available (in the public domain)
official link:	`http://www.ma.ic.ac.uk/~jcash/IVP_software/mebdftest/mebdfdae.f`
problems type:	ODEs and DAEs of index less than or equal to 3
IVPtestset files:	solver: mebdfdae.f
	driver: mebdfd.f
	auxiliary files: the linear algebra routines are included in `medbdfdae.f`.

4.2 Numerical method

The code MEBDFDAE uses the Modified Extended Backward Differentiation Formulas of Cash, that increase the absolute stability regions of the classical BDFs [Cas79, Cas83, Cas03, Hin83, HW96]. These methods are A-stable up to the order 4 and stiffly stable for orders up to 9; therefore they are especially suited for the solution of stiff systems of ODEs [CC92]. The orders of the implemented formulae range from 1 to 8.

4.3 Implementation details

The formulae implemented are three-stages general linear methods with the same Jacobian to be used in the Newton iteration for all the stages. Blas and Lapack auxiliary routines are also used. Versions of this solver for the solutions of ODEs are MEBDF and MEBDFSO, the last one is designed to solve stiff Initial Value Problems for very large sparse systems of ODEs, where the linear equation solver is replaced by the sparse solver YSMP [EGSS77]. Extensions of MEBDFDAE for the solution of very large sparse systems of DAEs is given by the solver MEBDFSD, where the sparse solver used is MA28 [I.S77]. A MATLAB translation of MEBDFDAE is available at the official link `http://www.ma.ic.ac.uk/~jcash/MATLAB_software/MEBDF.m`.

4.4 How to solve test problems with MEBDFDAE

Compiling

```
f90  -o dotest  mebdfdaed.f  problem.f  mebdfdae.f   report.f,
```

will yield an executable `dotest` that solves the problem, of which the Fortran routines in the format described in Section IV.3 are in the file `problem.f`.

As an example, we perform a test run, in which we solve problem HIRES. Figure I.4.1 shows what one has to do.

References

[Cas79] J. Cash. *Stable Recursions with applications to the numerical solution of stiff systems.* Academic Press, New York, 1979.

[Cas83] J. Cash. The integration of stiff initial value problems in o.d.e.s using modified extended backward differentiation formulae. *Comp. and Maths. with Applics.*, 9:645–657, 1983.

```
$ f90 -O5  -o dotest mebdfdaed.f hires.f mebdfdae.f   report.f
$ dotest

  Test Set for IVP Solvers (release 2.3)

  Solving Problem HIRES using MEBDFDAE

 User input:

 give relative error tolerance:
1d-4
 give absolute error tolerance:
1d-4
 give initial stepsize:
1d-4

 Numerical solution:

                                                    scd
       solution component             ---------------------------    ignore
                                      mixed       abs        rel    mix - abs,rel
 ----------------------------------   -----      -----      -----   -------------
 y(  1) =  0.7324251767207330E-003     5.33       5.33       2.19
 y(  2) =  0.1433221554010029E-003     6.03       6.03       2.19
 y(  3) =  0.5800420518076766E-004     6.05       6.05       1.82
 y(  4) =  0.1166962417102632E-002     5.06       5.06       2.13
 y(  5) =  0.2241753919183594E-002     3.84       3.84       1.22
 y(  6) =  0.5760280012688669E-002     3.32       3.32       1.12
 y(  7) =  0.2767358761415102E-002     4.08       4.08       1.54
 y(  8) =  0.2932641238585708E-002     4.08       4.08       1.54

 used components for  scd                 8          8          8
 scd of Y (maximum norm)               3.32       3.32       1.12


 using mixed error yields mescd        3.32
 using relative error yields scd                             1.12

 Integration characteristics:

    number of integration steps        97
    number of accepted steps           94
    number of f evaluations           168
    number of Jacobian evaluations     21
    number of LU decompositions        21

 CPU-time used:                       0.0020 sec
```

FIGURE I.4.1: *Example of performing a test run, in which we solve problem HIRES with MEBDFDAE. The experiment was done on an ALPHAserver DS20E, with a 667MH EV67 processor. We used the Fortran 90 compiler* `f90` *with the optimization flag* `-O5`.

[Cas03] J. Cash. Efficient numerical methods for the solution of stiff initial-value problems and differential algebraic equations. *Proc. Roy. Soc. London, A*, 459:797–815, 2003.

[CC92] J. Cash and S. Considine. An mebdf code for stiff initial value problems. *Acm Trans Math Software*, pages 142–158, 1992.

[EGSS77] S.C. Eisenstat, M.C. Gursky, M.H. Schultz, and A.H. Sherman. Yale sparse matrix package ii. the nonsymmetric codes. Technical Report 114, Department of Computer Science, Yale University, New Haven, CT, 1977.

[Hin83] Alan C. Hindmarsh. ODEPACK, a systemized collection of ODE solvers. In R. Stepleman et al., editors, *Scientific Computing*, pages 55–64, Amsterdam, 1983. IMACS, North-Holland Publishing Company.

[HW96] E. Hairer and G. Wanner. *Solving Ordinary Differential Equations II: Stiff and Differential-algebraic Problems*. Springer-Verlag, second revised edition, 1996.

[I.S77] I.S.Duff. Ma28-a set of fortran subroutines for sparse unsymmetric linear equations. Technical report, Technical Report AERE-R8730, Harwell, 1977.