8 Solver RADAU5

8.1 General information

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last update:	January 18, 2002
language:	Fortran 77
availability:	the code RADAU5 is freely available (in the public domain)
official link:	`http://www.unige.ch/~hairer/prog/stiff/radau5.f`
problem type:	ODEs and DAEs of index less than or equal to 3
IVPtestset files:	solver: radau5.f
	driver: radau5d.f
	auxiliary files: radaua.f (auxiliary linear algebra routines)

8.2 Numerical method

The code RADAU5 uses an implicit Runge-Kutta method (Radau IIa) of order 5 (three stages) with step size control and continuous output. It is written for problems of the form $My' = f(t, y)$ with a possibly singular matrix M. It is therefore also suitable for the solution of differential-algebraic problems.

8.3 Implementation details

Nonlinear systems are solved by a simplified Newton iteration. A similarity transformation on the inverse of the Butcher array is performed in order to reduce the computational cost associated to the solution of linear systems (see [HW96], page 121) so that, each time the Jacobian is updated, a factorization of one real and one complex matrix of the same dimension as that of the continuous problem is needed.

8.4 How to solve test problems with RADAU5

Compiling

```
f90  -o dotest  radau5d.f  problem.f  radau5.f  radaua.f  report.f,
```

will yield an executable `dotest` that solves the problem, of which the Fortran routines in the format described in Section IV.3 are in the file `problem.f`.

As an example, we perform a test run, in which we solve problem HIRES. Figure I.8.1 shows what one has to do.

References

[HW96] E. Hairer and G. Wanner. *Solving Ordinary Differential Equations II: Stiff and Differential-algebraic Problems.* Springer-Verlag, second revised edition, 1996.

```
$ f90 -O5 -o dotest radau5d.f hires.f radau5.f radaua.f report.f
$ ./dotest

  Test Set for IVP Solvers (release 2.3)

  Solving Problem HIRES using RADAU5

 User input:

 give relative error tolerance:
1d-4
 give absolute error tolerance:
1d-4
 give initial stepsize:
1d-4

 Numerical solution:

                                                     scd
        solution component            ---------------------------     ignore
                                       mixed       abs        rel    mix - abs,rel
 ----------------------------------    -----      -----      -----   -------------
 y(  1) =  0.7485152484440879E-003      4.94       4.94       1.81
 y(  2) =  0.1464912389469645E-003      5.65       5.65       1.81
 y(  3) =  0.6101426280653334E-004      5.67       5.67       1.44
 y(  4) =  0.1196763210067838E-002      4.68       4.68       1.75
 y(  5) =  0.2731889907948499E-002      3.46       3.46       0.84
 y(  6) =  0.7347017643277632E-002      2.96       2.96       0.75
 y(  7) =  0.3074620885907540E-002      3.65       3.65       1.10
 y(  8) =  0.2625379114092413E-002      3.65       3.65       1.10

 used components for  scd                  8          8          8
 scd of Y (maximum norm)                2.96       2.96       0.75


 using mixed error yields mescd         2.96
 using relative error yields scd                              0.75

 Integration characteristics:

    number of integration steps        38
    number of accepted steps           31
    number of f evaluations           295
    number of Jacobian evaluations     20
    number of LU decompositions        36

 CPU-time used:                       0.0010 sec
```

FIGURE I.8.1: *Example of performing a test run, in which we solve problem HIRES with RADAU5. The experiment was done on an ALPHAserver DS20E, with a 667MHz EV67 processor. We used the Fortran 90 compiler* `f90` *with the optimization flag* `-O5`.